Math 213, Fall 2025, Final Exam
Read Carefully, Write Neatly, Sketch Accurately
There are 14 questions (see page 2 for 13-14 and formulas). Total of 120 points.

1. (4 points) Find the center and radius of the sphere given by:
$(x+3)^2+(y-1)^2+(z-2)^2=25$

2. (6 points) Find the vector projection of the vector $\mathbf{u}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ onto $\mathbf{v}=3\mathbf{i}+0\mathbf{j}+4\mathbf{k}$.

3. (6 points first part, 4 points second part, 10 points total)
Find the tangent line to the curve $\mathbf{r}(t)=t^2\mathbf{i}+(2t-1)\mathbf{j}+t^2\mathbf{k}$ when t=3.
Set up the integral for the length of the curve $\mathbf{r}(t)=t^2\mathbf{i}+(2t-1)\mathbf{j}+t^2\mathbf{k}$ from t=3 to t=5.
Do not evaluate the integral.

4. (8 points) If $x(y+z^3)=0$ defines a surface, find the equations of the tangent plane and the normal line (parametric equations) to the surface at the point (1, -8, 2).

5. (8 points) Find all local maxima, minima and saddle points of
$f(x,y)=x^2-2xy+2y^2-2x+2y+3$ and classify them.

6. (10 points) Find the point(s) on the curve $x^2y^2=81$ that are closest to the origin using Lagrange multipliers.

7. (6 points) Reverse the order of integration for the integral $\int_0^{\ln 2}\int_{e^y}^{2} dxdy$. Do not evaluate.

8. (10 points) Set up the integral for a function f as an iterated integral in cylindrical coordinates over the solid region D that is bounded below by the plane z=0, on the side by the cylinder $r=\cos\theta$ and on the top by the paraboloid $z=3(x^2+y^2)$. That is, give the limits of integration for the integral $\iiint_D f(r,\theta,z)\, rdzdrd\theta$. Sketch the curve where the cylinder intersects the xy-plane.

9. (10 points) Show that the differential form $\sin y\cos x dx+\cos y\sin x dy+dz$ is exact. Find the potential function and evaluate the line integral $\int_{(1,0,2)}^{(3,4,0)} \sin y\cos x dx+\cos y\sin x dy+dz$

10. (12 points) Find directly (without using Green's Theorem) the counterclockwise circulation for the field $\mathbf{F}=(x-2y)\mathbf{i}+x\mathbf{j}$ around the unit circle centered at the origin.

11. (6 points) Provide a parametrization of the form $\mathbf{r}(r,\theta)=f(r,\theta)\mathbf{i}+g(r,\theta)\mathbf{j}+h(r,\theta)\mathbf{k}$ of the surface cut from the elliptical paraboloid $z=x^2+4y^2$ by the planes z=0 and z = 4.

12. (6 points) Find the surface area differential $d\sigma$ for the parametrization $\mathbf{r}(x,z)=z\mathbf{i}+xz\mathbf{j}+x\mathbf{k}$

13. (12 points) Set up the surface integral (using the right hand side of Stokes' theorem) for the circulation of the field $\mathbf{F} = xy\mathbf{i} + z\mathbf{j} + xz\mathbf{k}$ (counterclockwise as viewed from above) around the boundary of the triangle in the first octant cut from the plane $x + 2y + 3z = 6$. Do not evaluate. (Your answer should be a double integral in terms of x and y.)

14. (12 points) Use the Divergence Theorem to find the inward flux of the field $\mathbf{F} = \frac{x^3}{3}\mathbf{i} + \frac{y^3}{3}\mathbf{j} + 2z\mathbf{k}$ across the boundary of the region bounded by the cylinder $x^2 + y^2 \leq 4$ and the planes $z=0$ and $z=3$

$x = r\cos\theta;\ y = r\sin\theta;\qquad x = \rho\cos\theta\sin\phi; y = \rho\sin\theta\sin\phi; z = \rho\cos\phi$

$$\iint_R f(x,y)\,dA = \iint_R f(x,y)\,dxdy = \iint_R f(r,\theta)\,rdrd\theta$$

$$\int\int\int_D f(x,y,z)\,dxdydz = \int\int\int_D f(r,\theta,z)\,r\,drd\theta dz =$$

$$\int\int\int_D f(\rho,\phi,\theta)\ \rho^2\sin\phi\ d\phi d\theta d\rho =$$

$$\int\int\int_D f(u,v,w)\,J(u,v,w)\,dudvdw \text{ where } J(u,v,w) = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix}$$

$$\int_C f(x,y,z)\,ds = \int_C f(x,y,z)\left|\frac{dr}{dt}\right|dt = \int_C f(x,y,z)\,|v(t)|\,dt$$

$$\int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{T}}ds = \int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int_C \vec{\mathbf{F}}\cdot\frac{d\vec{\mathbf{r}}}{dt}dt = \int_C Mdx + Ndy + Pdz = \int_C\left(M\frac{dx}{dt} + N\frac{dy}{dt} + P\frac{dz}{dt}\right)dt$$

$$\int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}ds = \int_C Mdy - Ndx = \int_C\left(M\frac{dy}{dt} - N\frac{dx}{dt}\right)dt$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{T}}ds = \iint\left(N_x - M_y\right)dxdy = \iint\left(\nabla\times\vec{\mathbf{F}}\right)\cdot\vec{\mathbf{k}}dxdy$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}ds = \iint\left(M_x + N_y\right)dxdy = \iint\left(\nabla\cdot\vec{\mathbf{F}}\right)dxdy$$

$$\vec{\mathbf{r}}(u,v) = f(u,v)\,\vec{\mathbf{i}} + g(u,v)\,\vec{\mathbf{j}} + h(u,v)\,\vec{\mathbf{k}}$$

$$\iint d\sigma\text{:} \iint\left|\vec{\mathbf{r}}_u\times\vec{\mathbf{r}}_v\right|dudv;\ \iint\frac{|\nabla F|}{|\nabla F\cdot\vec{\mathbf{k}}|}dxdy;\ \iint\sqrt{1 + f_x^2 + f_y^2}\,dxdy$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}d\sigma = \iint\vec{\mathbf{F}}\cdot\left(\vec{\mathbf{r}}_u\times\vec{\mathbf{r}}_v\right)dudv$$

$$\oint \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \iint\left(\nabla\times\vec{\mathbf{F}}\right)\cdot\vec{\mathbf{n}}d\sigma;\qquad \iint\vec{\mathbf{F}}\cdot\vec{\mathbf{n}}d\sigma = \int\iint\left(\nabla\cdot\vec{\mathbf{F}}\right)dV$$